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A Review of the Economic Foundations of Earnings and Discounting Theories Used in Forensic Economics

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Abstract

In this paper, we review the economic foundations for earnings and discounting procedures used in forensic economics. The purpose of this unified presentation of generally accepted economic principles is to (a) provide documentation for the common economic conclusions derived by forensic economists, and (b) advance the appreciation and acceptance of the science of forensic economics. The format of the presentation is that the economic concept or principle is stated. The statement is followed by the commonly accepted economic theory that supports the principle. A brief discussion of the implications follows the theoretical presentation. Following the conclusion is a comprehensive list of references to the published literature that support the economic theories contained within the paper.

Introduction

In this paper, we review the economic foundations for earnings and discounting procedures used in forensic economics. The purpose of this unified presentation of generally accepted economic principles is to (a) provide documentation for the common economic conclusions derived by forensic economists, and (b) advance the appreciation and acceptance of the science of forensic economics.

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This paper is organized in two parts. The first part contains the economic theories that provide the foundation for estimating expected capacity and earnings. The first part contains six sections: 1) Earnings Capacity at a Point in Time; 2) Growth in Earning Capacity; 3) Earning Capacity and Labor Supply; 4) Nominal and Real Growth in Earning Capacity; 5) Boundaries of Expected Growth in Earning Capacity; and, 6) Estimating Expected Earning Capacity and Expected Earnings. The second part of the paper provides the economic foundation for interest rate determination and present value calculations. The second part contains six sections: 1) The Economic Foundation of the Real Interest Rate; 2) Discounting Formula; 3) Risk Associated with Receiving the Future Value; 4) Inflation and the Discount Rate; 5) Risk Associated with the Investment Vehicle; and, 6) Evaluating the Selection of a Discount Rate. The format of the presentation is that the economic concept or principle is stated. The statement is followed by the commonly accepted economic theory that supports the principle. A brief discussion of the implications follows the theoretical presentation. While the economic theory sections are lengthy in comparison to the principles themselves, the length is necessary to establish common discussion points among experts addressing the forensic economics principles. Following the conclusion is a comprehensive list of references to the published literature that support the economic theories contained within the paper.

Part One. Expected Earning Capacity and Earnings

It is first useful to present short definitions of expected earning capacity and earnings. An individual's *earning capacity* is equal to the amount of money he or she could reasonably expect to earn working at a job that fully utilizes his or her abilities. An individual's *expected earnings* are the money amounts it is anticipated that he or she will actually earn working at a job. The extent to which the person utilizes his or her abilities in the labor market over the course of working life determines his or her *lifetime expected earnings.*

Earning Capacity at a Point in Time

Economic theory holds that a plaintiff's earning capacity at a point in time (for example, at an injury or death date) is determined by the employers who know the abilities of the plaintiff. The earning capacity wage rate is based on the additional revenue earned by the employer by hiring the plaintiff's labor. The factors influencing the plaintiff's earning capacity are the prices of the

employers products, technology utilized in the firm, the plaintiff's innate abilities, and the amount of human capital the plaintiff possesses.

Economic Theory

The generally accepted economic theory of earning capacity begins with the wage rate[1] conditions of a profit-maximizing firm (or organization) in economic equilibrium. It is uniformly accepted that when a firm maximizes profits and is in equilibrium, a laborer working at the firm receives a wage rate, w, which is equal to the marginal revenue product of his or her labor input, MRP_L. We write this relationship as:

$$w = MRP_L, \tag{1}$$

where MRP_L is defined as $\partial R / \partial L$, or the partial derivative of R (the firm's revenue) with respect to L (labor input), i.e. the change in revenue when labor changes with all other variables in the specified profit maximizing economic function held constant. So, in the case of MRP_L, economists are interested in the change in firm's revenue associated with a change in the amount of labor employed with all other production factors (e.g., capital, land) remaining the same.

When studying profit maximization, an important economic concept is marginal revenue, MR, or the change in firm's revenue associated with a change in the firm's output, P. Using the established notation,

$$MR = \partial R / \partial P. \tag{2}$$

With mathematical manipulation, we can incorporate equation (2) into the right-hand-side of equation (1) as:

$$MRP_L = \partial R / \partial L = \partial R / \partial P \times \partial P / \partial L = MR \times \partial P / \partial L. \tag{3}$$

The condition $\partial P / \partial L$ represents the marginal change in output with a change in the amount of labor employed, or simply, the marginal product of labor, MP_L. We can now rewrite equation (3) as:

$$MRP_L = MR \times MP_L. \tag{4}$$

Substituting the right-hand-side of equation (4) into equation (1), economists accept that the product of marginal revenue and the marginal product of labor is the determinant of the wage rate:

$$w = MR \times MP_L. \tag{5}$$

Equation (5) summarizes the economic theory that the wage rate available to an individual (i.e., his or her earning capacity) from a profit-maximizing firm in equilibrium is equal to the product of marginal revenue and the marginal product of the individual's labor. Although there may be firm-specific exceptions to this relationship (created by things such as monopsony in the market for labor, efficiency wages, discrimination, or that wage rates may vary randomly around MRP_L due to mistakes), economists generally accept that the transferability of assets ensures that firms eventually return to the position of relating wages to marginal revenue product of labor. Additionally, in the short-term, institutional forces may affect the model's assumptions of a profit maximizing firm and a competitive labor market; however, economists generally accept that institutions over the long-term are unable to significantly separate the relationship of the wage rate to the marginal revenue product of labor.[2] If for some extenuating circumstances where marginal revenue product does not determine an individual's wage rate, plausible economic explanation within the framework of the theoretical positions is required.

The firm's revenue, R, is equal to the product of its output, P, and the price of its output, y:

(6) $$R = P \times y.$$

Marginal revenue is defined as:

(7) $$MR = \partial R / \partial P = y + \partial y / \partial P \times P$$

and changes in marginal revenue will occur only when the price of output, y, changes. The price change will be the result of either a relative price change or a change in the price level (inflation). The laborer has no control over the market price of the product he or she produces (i.e., a laborer does not influence price inflation or, presumably, relative prices). Therefore, the wage rate that a laborer receives has determining conditions through a firm's marginal revenue that are exogenous to the quality or kind of work supplied by the laborer to the firm. Future relative price changes between firm-specific price levels and aggregate consumption good price levels are generally assumed zero or embodied within the empirical observations of industry or occupation wage change data. There are two reasons for this statement. First, individual skills are usually not so specific that a worker is permanently confined to work in one particular industry. If the relative prices in an industry decline resulting in a decreased wage rate, individuals

are capable of transferring their labor to a different industry absent that impact on the wage rate. In this regard, the change in relative prices has limited consequences on wage rates.[3] The other reason is that changes in relative prices are difficult to forecast in isolation. Because of the limited relevance of a change in relative prices to wage rates and the difficultly of forecasting a change in relative prices, a specific forecast of no change in relative prices is standard in most cases.

The marginal product of labor, how much the firm's output changes as it changes its amount of labor input, depends upon the specific characteristics of the labor supplied by the individual to the firm. Economists analyze the individual's productivity from three viewpoints: (1) individual innate ability; (2) individual level of human capital; and, (3) the existing technological conditions within the economy (each discussed below). Hence, an individual's earning capacity is dependent upon productivity factors unique to themselves (innate ability and human capital) and productivity factors that apply to all labor in the economy (existing technology). Changes in any one of these factors will change wage rates (earning capacity), other things constant. In any particular case, the economist addresses these factors when forming estimates of the ability of a person to attain and maintain an anticipated level of earning capacity and growth.

From equations (3) – (5), when $\partial R/\partial L > 0$ firms desire to employ labor and workers have a positive earning capacity, $w > 0$.[4] Firms hire workers with attributes A. Since all persons are not homogeneous in A, they contribute differently to firm revenue and they have different earning capacities. Some workers with low marginal product due to disability, few innate skills, or low human capital can cause $\partial R/\partial L \leq 0$ and firms will not demand that worker's labor, so earning capacity becomes zero. For example, an employer's expense to accommodate or supervise a disabled worker might be greater than the revenue achieved with the addition of that disabled worker to their production function. In other cases, the worker's marginal product might be at a low level and satisfy $\partial R/\partial L > 0$ but for a limited number of jobs or hours in a day, week, or year. For example, a person might be limited due to disabilities to a few rare jobs that could not be reliably counted upon (e.g., stamp-licker for three hours per month). Also, in the case where the worker's marginal revenue product per hour is less than minimum wage, legal constraints prevent a firm from hiring that worker.

Evaluation Discussion

Under a common assumption of competitive organizations and labor markets, the role of prices and technologies are minimized in earning capacity determination within a forensic economic setting. Therefore, the forensic economist should provide particular focus on the innate abilities and human capital characteristics of the plaintiff when presenting earning capacity forecasts. Innate abilities refer to the plaintiff's natural mental and physical capacities. Human capital refers to skills and positions acquired by the plaintiff throughout his or her life.

Forensic economists usually do not determine the natural mental and physical capacities of the plaintiff, nor do they usually take inventories of the skills acquired by the plaintiff. Vocational and psychological experts are relied upon to test the plaintiff on those qualities when a clear record of attainment is not apparent. The forensic economist matches the plaintiff's demonstrated levels of innate ability and human capital to empirical labor market data. For example, if a plaintiff has the necessary innate ability and human capital to be a warehouse fork lift operator, the forensic economist matches reliable empirical economic data regarding warehouse fork lift operators to the remaining worklife of the plaintiff. The evaluator should query the forensic economist for his or her consideration of the plaintiff's innate ability and human capital that comprises the estimate of the plaintiff's earning capacity, because there should be a direct correspondence. For example, if the plaintiff's abilities do not reasonably match with those necessary for sustainable, gainful employment as a loading dock worker, then the legal *Daubert* or *Frye* evaluator should declare that forming the plaintiff's earning capacity on the empirically recorded earnings of loading dock workers violates generally accepted economic principles.

Growth in Earning Capacity

Not only must a forensic economist forecast earning capacity at a point in time in a plaintiff's working life, it is also necessary to forecast future changes in earning capacity by age or employment tenure recognizing changes in price levels (inflation), technology, innate ability, and human capital. Under the common assumption of the plaintiff's labor being able to transfer from competitive organization to competitive organization, the plaintiff's earning capacity will change by the price levels in the market he or she is contributing labor. For example, a 5 percent increase in the price level will result in a 5 percent increase in the wage rate, other things equal. Changes in technology enable laborers to being able to contribute more to the

organization's revenue, other things equal, thereby increasing their earning capacity. Innate abilities generally decline with age and the loss of innate ability also causes earning capacity to decline (the decline can be gradual with advancing age or immediate with the onset of disability). Human capital is also subject to change with age. In general, human capital provides substantial growth in earning capacity early in life as skills are obtained, then levels off as a plaintiff becomes settled into a career, and then late in work life declines as acquired skills and positions become dated to those of other younger competing laborers, other things equal.

Economic Theory

When studying the changes in the wage rate (or earning capacity) over time, economists frame their models within the total differential of equation (5) with respect to time, t, as follows:

$$\frac{dW}{dt} = MP_L \times \frac{dMR}{dt} + MR \times \frac{dMP_L}{dt}. \tag{8}$$

Changes in MR over time are a function of changes in relative price levels and changes in the price level (inflation). The relationship of the changes in the price of the firm's products to changes in marginal revenue, according to equation (7), is such that a particular percentage change in the price of the product, other things equal, will result in that same percentage change in marginal revenue. Then according to equation (8), that percentage change in marginal revenue will translate into that same percentage change in the wage rate. For example, a 5 percent increase in the equilibrium price level will result in a 5 percent increase in the wage rate, other things equal.[5] The changes in an individual's MP_L over time are a function of changes in innate abilities, human capital, and technological conditions. Innate abilities refer to the individual's natural mental and physical capacities that are subject to changes with age. Loss of innate ability creates negative marginal productivity growth causing earning capacity to decline (the decline can be gradual with advancing age or immediate with the onset of disability).

Human capital, in contrast to innate abilities, refers to skills and positions acquired by an individual throughout life. Economic theory shows that, in general, human capital is acquired early in life leading to a concave behavior of wage rates over the life cycle, *ceteris paribus*. The generally accepted Ben-Porath (1967) human capital model contains the derivation of the generally

accepted concave life-cycle earnings pattern. We summarize the Ben-Porath model as follows:

(9) $$E_t = wC_t$$

where,
E_t is earning capacity,
w is the wage rate per unit of human capital stock, and
C_t is the number of units of human capital processed in time t.

The behavior of E_t over time will depend on the behavior of C_t over time. Increasing C_t involves an investment cost that requires the use of some portion of the existing human capital stock (i.e., the human capital stock used for investment could otherwise be used for current earnings). The individual benefit to increasing C_t is an increase in remaining lifetime earning capacity. By equating the marginal cost and marginal benefit of increasing the human capital stock, the human capital model shows that the investments will be greater at younger ages and then diminish with resulting in the concavity of wage rates (or earning capacity) over a worker's lifetime. To address human capital's impact on earning capacity, economists on a case-by-case basis empirically construct data for life-cycle earnings patterns that coincide with the theoretical models of human capital.

As an example of the life cycle earning pattern, consider a male high school graduate. His real wage rate increases (wage rate growth unrelated to product price changes) would be expected to grow rapidly at the beginning of his work-life as he is increasing his human capital through on-the-job training. This on-the-job training increases his productivity (MP_L) and thus makes him more valuable to his employer and his wage rate increase accordingly. At some point, the increases in human capital resulting from on-the-job training would begin to level off as he learns what there is to learn and is less willing to devote time to increasing human capital as the time period for investment payoff decreases. Then there is an expected decrease in earning capacity toward the end of his worklife due to depreciation in human capital acquired earlier in life (earning capacity will also decrease toward the end of worklife due to a reduction in innate abilities such as stamina and strength as discussed above).

Technological change can also result in changes in the individual's MP_L and thereby his or her relative wage rate (or earning capacity). Labor-using technological progress implies that at a constant capital-labor ratio, the marginal rate of technical substitution of capital for labor increases. This would result in an increasing relative wage rate and an increase in labor's

share of output. This result follows from the standard model. By definition, economists recognize technology conditions as:

$$MRTS = \frac{MP_L}{MP_K} \tag{10}$$

where,
$MRTS$ is the marginal rate of technical substitution (capital for la bor),
MP_L is the marginal productivity of labor,
and,
MP_K is the marginal productivity of capital.

An equilibrium condition for profit maximization is

$$\frac{MP_L}{MP_K} = \frac{w}{k} \tag{12}$$

where k is the per unit price of capital. A labor-using technological change is an increase in technology such that at a constant capital-labor input ratio, $\frac{K}{L}, \frac{MP_L}{MP_K}$ increases. Thus, if there is a labor-using technological change, w increases relative to k. It may be the case that payments to labor have increased relative to payments to capital due to technological changes implying a secular increase in wage rates. On a case-by-case basis, an economist may have evidence that this increase is expected to occur as supported by the theoretical models of technological change. Additionally, as we point out, there may be, but not always necessarily be, other components of expected wage rate determination and growth such as secular or efficiency wage trends, institutional influences, or persistent monopsony.

Earning Capacity and Labor Supply

Generally accepted theoretical models of labor supply are based on consumer demand for consumption. A consumer divides a fixed amount of allocable time, T, into hours worked in the labor market, h, and hours spent at consumption, l. Ignoring savings, the consumer with characteristics A possesses a utility function defined by the consumption of commodities, x, and hours of work, h, as:

$$U = U(x, h; A, \varepsilon) \tag{13}$$

where ε stands for the consumer's tastes, availability for home production, or whatever essential item that the consumer balances between work in the labor market and consumption. The partial derivative of U in equation (13) with respect to x is assumed to be positive (i.e., utility increases with consumption) and negative with respect to h (i.e., utility declines with work usually strongly beginning somewhere in the neighborhood of normal hours spent at work). Income from market work depends upon hours of work h and, in the simplest case, the fixed wage rate w (or earning capacity) per unit of work related to equation (5). If p is the fixed per unit price of the bundle of commodities x, and if y is income independent of the labor supply decision, then the consumer's budget constraint is:

(14) $$px = wh + y$$

The consumer is assumed to do the best he or she can, so he or she chooses $x > 0$ and $h \geq 0$ that maximizes the utility equation (13) subject to the budget constraint (14). When utility maximization leads to a positive number of hours of work, $h > 0$, the first-order condition equalizes the real wage (w/p) and the negative of the marginal rate of substitution of working hours for commodities as:

(15) $$\frac{w}{p} = \frac{\partial U/\partial h}{\partial U/\partial x}.$$

Putting together equations (13) – (15), the real wage is the consumer's implicit value of his or her time at the margin between participating and not participating in the labor market. If at the margin, the market's valuation of the consumer's time, w, exceeds the consumer's implicit value of his or her time when $h = 0$, w^* or the reservation wage, then the consumer will participate in the labor market and supply a positive number of hours h of market work. Conversely, if at the margin the consumer places a greater value on an extra unit of his consumption time than does the market on his or her work time ($w^* > w$), then the consumer will not supply labor to the labor market.

Nominal and Real Growth in Earning Capacity

Economists may use a variety of methodologies to estimate the principle components of earning capacity and growth, but they may not reject the theoretical basis of equations (1)-(12) and still be in conformity with generally accepted economic theory. When estimating expected wage rate growth, we see from equations (5) and (8) that an economist can separate his or her

analysis of the reasons wage rates change into marginal revenue considerations and marginal product considerations. When an economist includes changes in the price level (inflation) as one of the reasons for wage rate changes, he or she is said to be studying "nominal rates" of wage rate growth. When an economist does not include changes in price levels as one of the reasons for wage rate changes and focuses only on relative prices and marginal product, he or she is said to be studying "real rates" of wage rate growth.

Nominal rates of wage rate growth estimate future-dollar levels of wage rates while real rates of wage rate growth estimate constant-dollar levels of wage rates. Economists accept nominal and real rates of wage rate growth as long as the economist consistently expresses all terms within his or her forecast as nominal or real. For example, an economist may use future nominal expected wage rate growth to calculate a forecast of future-dollar levels of earnings and then discount to present value using a future expected nominal rates of interest. Or, an economist may use future real expected wage rate growth to calculate a forecast of current-dollar levels of future earnings and then discount to present value using a future expected real rate of interest. The economist cannot use combinations of nominal growth/real interest nor real growth/nominal interest.

When working in "real terms," the economist estimates future wage rates absent price inflation. Whatever the exogenous level of price inflation may be during the base year, t_0, and any other year, t_0+n, in the future, the economist's real dollar forecast translates to nominal money wage rates in year t_0+n that are $\{[(1+\text{inflation}) \times (1+\text{real wage rate growth})]^n - 1\}$ percent higher than the nominal level of money wage rates in the beginning year t_0.

Boundaries of Expected Growth in Earning Capacity

There are no theoretical conditions that constrain the components of expected wage rate growth as developed through equations (1)-(12), at any future time t, to be less than zero, equal to zero, or greater than zero. Hence, expected wage rate growth (in nominal or real terms) is unbounded: it can be less than zero, equal to zero, or greater than zero (or, the first-order conditions of equation (5) are unconstrained). Additionally, there are no theoretical conditions that constrain the functional shape of expected wage rate growth (e.g. expected wage rate growth can be decreasing, constant, or increasing). For example, economists know human capital growth to be

concave, so absent an equalizing convexity in the other components of expected wage rate growth, we expect non-constant wage rate growth when human capital is a directly measured component of forecasted wage rate growth.

The absence of theoretical constraints to expected wage rate growth does not give theoretical and methodological equality to all estimates of expected wage rate growth. Any expected wage rate growth estimated for an individual must have sound theoretical, methodological, and empirical foundations based within the generally accepted economic principles. While debate can take place regarding methodologies and empirical estimates, the generally accepted theoretical or underpinning positions always hold as the fundamental basis for estimating expected wage rate growth.

Estimating Expected Earning Capacity and Expected Earnings

At a point in time, generally accepted economic theory supplies the framework to evaluate an individual's earnings capacity through his or her marginal product and earnings through his or her demand for consumption. The economic situation of a plaintiff at a point in their lifetime, which we call an economic "state," is caused by an aggregate of factors some of which are specific to the plaintiff (e.g., innate ability, human capital, and choice of leisure versus work) and others that are relevant to all persons in the economy (e.g., price levels and technology).

We have shown that generally accepted economic theory includes components that anticipate transition or movement from one economic state to another during the course of a lifetime. For example, the decrease in innate ability with age (e.g., physical deterioration) decreases the individual's marginal product capability leading to a decrease in his or her earning capacity, other things equal, which influences his or her demand for consumption which controls how much labor he or she supplies to the market which ultimately results in determining his or her earnings level. Following this example, we limit the analysis to two states, before and after the decline in innate ability (other thing equal), labeled S_1 and S_2 respectively. From generally accepted economic theory, we know that per unit earning capacity in S_1 will be greater than per unit earning capacity in S_2; however, the impact on actual dollar earnings is unpredicted from theory and requires estimation based on the individual's presumed decision-making when faced with a reduced per unit earning capacity due to the loss of innate ability.

Some people might substitute more hours of work to maintain the same total earnings in S_2 that they had in S_1 while others might choose to retire from the labor force in S_2 because the per unit value of their consumption time exceeds S_2.

In order to estimate future multi-year earning capacity and expected earnings, economists use econometric models that exist on the basis of generally accepted economic theories. If an economist does not bind his econometric estimation of the expected present value of multi-year earnings losses to economic theory, he is "able to prove anything at any moment he likes. But then, he is telling stories, not making theory."[6] Using a variety of empirical data to properly account for the theoretical modeling of expected variability in lifetime earnings and earning capacity, the economist describes the relationships that produce earning capacity and earnings by presenting statistics. Going back to the human capital example, economists are interested in measuring the affects of human capital on lifetime earnings. In order to describe the relationship statistically, the economist obtains observations of the earnings of similar persons of type A across age or tenure in an occupation, called the "sample." The human capital economic theory model sets forth the hypothesis of the mathematical form expected in the relationship of earnings to age or tenure (which is the standard concave life-cycle earnings pattern). From the sample, using statistical methods, the economist tests whether the mathematical form of human capital theory is observed within the sample. The economist presents an estimate of earnings resulting from the statistical relationship of age or tenure to earnings and then tests whether that relationship is adequately and reliably described by using deviations (or error rates) between the earnings in the sample and the predicted mathematical concave life-cycle earnings pattern. Therefore, economists use econometrics to obtain quantitative values of the relationships posited in the parameters of an economic theory and test those values for reliability as operators of economic theory.

Individuals have identifying characteristics at the present time (e.g., current abilities) and in their history (e.g., track records of earnings and human capital attainment) and the economy possesses current and historical characteristics (e.g., inflation, unemployment, and the progress of technology) that affect earning capacity. Likewise, persons of comparable characteristics over a lifetime of ages (i.e. a cross-section of the current population) present a relevant sample from which to draw information regarding transitions or movements in economic states over a lifetime. In order to present reliable

economic information to the trier of fact regarding the expected present value of multi-year earnings losses, the economist uses econometric methods to apply all of the relevant parameters of earning capacity and earnings to appropriate relevant samples of population data within the economy. While the detail in sampling and statistical estimation is subject to debate and variation amongst economists, conformity to the economic theories of earning capacity and expected earnings is required in all statistical studies. For example, a valid projection of earning capacity includes both human capital and innate ability analysis. An economist cannot project earning capacity based on a human capital attainment over a lifetime without also considering the expected effects of the reduction in innate ability which reduces earning capacity over a lifetime.

Evaluation Discussion

A forensic economist must be able to reveal how he or she studied the effects of price inflation and technology on the growth in the plaintiff's earning capacity. In most situations, price and technology growth will be presented together as a constant growth amount.

Part Two. Interest Rate Determination and Discounting Procedures

After a forensic economist estimates an expected multi-period earnings loss, the stream of expected future earnings must be reduced to a present value lump sum for transfer from the defendant to plaintiff.[7] Present value lump sum payments comply with two legal requirements: (1) the plaintiff's damages must be paid-in-full, and (2) the defendant must be equitably treated because the plaintiff will be able to receive a secondary stream of investment income from the portion of the lump sum unnecessary to compensate for current losses. Generally accepted economic theory anticipates the known rewards and risks of investing lump sums that allows reliable present value calculations. Below, we address those theories regarding the discounting procedure to account for the time value of money and investment risk. The first section describes how consumers' preference for immediate consumption versus postponed consumption necessitates a positive real interest rate. The second section reviews the standard discounting formula. The next sections contain the conditions that affect the discount rate used in the discounting formula: the amount of risk to be borne

by the compensated entity, the certainty of the receipt of the future dollar amount, and how inflation and other financial risk affects dollar amounts received in the future. We conclude with the process of evaluation of the discount rate chosen by an economist for a particular purpose.

The Economic Foundation of the Real Interest Rate

In classical economic theory, the interest rate is determined by the supply of and demand for savings. Suppliers of savings demand a real return, or real interest rate, to compensate them for postponed consumption. The annual real interest rate is the number of units of consumption goods received in the next year, minus one unit, in exchange for one unit of consumption goods foregone this year.

To discover the basic properties of the real interest rate, i, consider a simple economy where all agents have the same preferences and endowments with total utility,[8] U, determined within the following two-period intertemporal utility function:

$$U = u(c_1) + \frac{u(c_2)}{1+\rho}, \tag{16}$$

where,
c_t = consumption choices in period t, t = 1 or 2;
$u(c_t)$ = utility derived from the consumption of c in period t; and,
ρ is the preferences discount rate, or the amount by which agents discount next period utility to this period utility.

We assume that the utility function $u(c)$ is continuously differentiable, strictly increasing and strictly concave, or that the marginal utility $u'(c)$ is strictly positive and strictly decreasing.[9] Economists accept that $\rho > 0$, which implies that agents derive more utility from the immediate consumption of a good relative to the utility derived from consuming the good at a latter time. It is the preference for immediate consumption that results in a positive real interest rate in equilibrium.

Using intertemporal utility maximization theory, economists derive the first order condition for equilibrium. To determine the marginal rate of substitution between consumption in the first and second period, we take the total derivative of equation (16) as:

(17) $$u'(c_1)dc_1 + \frac{u'(c_2)}{1+\rho}dc_2 = 0,$$

and then rearrange to the common form of the marginal rate of substitution (*MRS*),

(18) $$\frac{dc_2}{dc_1} = MRS_{c_1c_2} = \frac{u'(c_1)}{\left(\frac{u'(c_2)}{1+\rho}\right)}.$$

An agent's objective is to maximize utility, equation (16), subject to their budget constraint. We assume that all agents can borrow or lend in the first period at the real interest rate, i. Let y_t be equal to real income the individual receives in period t. Then, as all income must be consumed by the end of the second period in our simple model, the intertemporal budget constraint is written as:

(19) $$c_1 + \frac{c_2}{1+i} = y_1 + \frac{y_2}{1+i}.$$

The consumer will maximize (16) subject to the constraint of (19). The maximization problem is

(20) $$\max_{\{c_1,c_2\}}\left[u(c_1) + \frac{u(c_2)}{1+\rho}\right]$$
$$\text{subject to } c_1 + \frac{c_2}{1+i} = y_1 + \frac{y_2}{1+i}$$

This maximization problem is solvable by using the Lagrangian multiplier method. First, we rearrange the budget constraint so that the right hand side is equal to zero: $c_1 + \frac{c_2}{1+i} - y_1 - \frac{y_2}{1+i} = 0$. Second, we write the Lagrangian function as:

(21) $$L(c_1, c_2, \lambda) = u(c_1) + \frac{u(c_2)}{1+\rho} - \lambda\left(c_1 + \frac{c_2}{1+i} - y_1 - \frac{y_2}{1+i}\right).$$

We then take the derivatives of the Lagrangian function with respect to c_1, c_2, and λ, and set them equal to zero:

$$u'(c_1) - \lambda = 0$$

$$\frac{u'(c_2)}{1+\rho} - \frac{\lambda}{1+i} = 0 \tag{22}$$

$$c_1 + \frac{c_2}{1+i} - y_1 - \frac{y_2}{1+i} = 0.$$

Using the first two equations of (22), substituting $u'(c_1)$ for λ, we obtain the equation

$$\frac{u'(c_2)}{1+\rho} - \frac{u'(c_1)}{1+i} = 0. \tag{23}$$

Rearranging (23), we obtain the first order condition

$$\frac{u'(c_1)}{\left(\frac{u'(c_2)}{1+\rho}\right)} = 1+i, \tag{24}$$

which describes how agents optimally choose c_1 relative to c_2 when they face the real interest rate i.

Recall that marginal utility $u'(c)$ is strictly decreasing so that the higher is c, the lower is $u'(c)$. Therefore, when the real interest rate is high, agents choose consumption in the two periods so that $u'(c_1)$ is high with respect to $u'(c_2)$, that is they choose a low value of c_1 relative to c_2. This result is intuitive: a high real interest rate encourages agents to shift consumption from the first period to the second; also, for a given interest rate, with an increase in the preferences discount rate, ρ, c_1 will increase relative to c_2. If $\rho = 0$ then the left hand side of equation (24) would equal one in equilibrium as the price of the product is unchanged. The accepted proposition that $\rho > 0$ results in an equilibrium real interest rate $i > 0$.

In equation (24), the real interest rate, i, is taken as exogenous where agents choose consumption based upon a given value of the real interest rate. However, in equilibrium, the real interest rate adjusts so that all markets clear. In our simple intertemporal model, agents have the same preferences and endowments and they have the same access to markets. For equilibrium to exist, $c_1 = y_1$ and $c_2 = y_2$. If we substitute the equilibrium condition into the first-order condition shown in equation (24), we obtain the equilibrium

pricing equation which determines the equilibrium real interest rate as a function of the agents' endowments and preferences:

$$\frac{u'(y_1)}{\left(\frac{u'(y_2)}{1+\rho}\right)} = 1+i, \tag{25}$$

According to equation (25), when the aggregate supply of consumption goods in the first period is low relative to the second, the real interest rate must be high to encourage agents to shift consumption from the first period to the second so that the aggregate demand for goods equals the aggregate supply in both periods. In addition, for a given set of endowments, the higher the preferences discount rate, ρ, the higher the real interest rate becomes. This result happens because when agents discount future utility at a high rate, they would like to shift consumption from the second period to the first, so a high real interest rate is needed to counterbalance this effect so that aggregate demand for goods equals the aggregate supply for both periods.

Discounting Formula

The formula used to calculate the amount required today, the present value, in order to compensate for an amount that would have been received in the future, the future value, is

$$PV = FV\left[\frac{1}{(1+i)^t}\right], \tag{26}$$

where,
PV = the present value,
FV = the future amount to be received in period t,
i = the interest rate, per time period, used for discounting,
and,
t = the number of time periods in the future until FV would be received.
The term in the brackets is often referred to as the discount factor and the interest rate, i, is often referred to as the discount rate. If there are several periods in the future where an amount would have been received, the formula is written as:

$$PV = FV_1\left[\frac{1}{(1+i)^1}\right] + FV_2\left[\frac{1}{(1+i)^2}\right] + \cdots + FV_n\left[\frac{1}{(1+i)^n}\right], \tag{27}$$

or, using summation notation,

$$PV = \sum_{t=1}^{n} FV_t \left[\frac{1}{(1+i)^t} \right], \tag{28}$$

where,
FV_t = the future amount to be received in period t.
The amount of future value, FV_t, is not necessarily the same amount at every t. Likewise, it is not required that the interest rate, i, has to be the same for every time period t.

Risk Associated with Receiving the Future Value

One factor that affects the choice of appropriate discount rate is the known probability (risk[10]) that is associated with receiving the future value. The greater the probability that the future amount would have been received, the greater is the present value of the amount, other things equal. Assume that an amount FV will be received with certainty in one year. Moreover, further assume that the appropriate risk free discount rate has been determined to be i percent so that, applying equation (26) the present value is

$$PV = FV \left[\frac{1}{(1+i)} \right]. \tag{29}$$

Now, assume that the amount FV will be received with the probability of η_{FV} where,
$0 \leq \eta_{FV} \leq 1$ so that the expected[11] amount received is defined by

$$E(FV) = FV \times \eta_{FV}. \tag{30}$$

With the existence of risk, it is not appropriate to calculate the present value of FV by using the risk free discount rate of i and amount FV as in equation (29). There are two equivalent methods of calculating the present value of FV when FV will be received with a probability of less than one. The discount rate of i can be used with the expected future value, $E(FV)$, so that the present value is

$$PV = E(FV) \left[\frac{1}{(1+i)} \right]. \tag{31}$$

The second method uses a discount rate that incorporates the probability of receiving the amount. The discount rate that incorporates the probability of receiving the amount FV is represented as, i_p, where

$$i_p = \left[\frac{(1+i)}{\eta_{FV}} \right] - 1, \tag{32}$$

then,

$$PV = FV\left[\frac{1}{(1+i_p)}\right]. \tag{33}$$

Substituting for i_p from equation (32) into equation (33) shows the equivalence of equations (31) and (33). There are situations when $E(FV)$ and i_p are each present in the discounting problem. When $E(FV)$ and i_p and are independent of each other and can be separately estimated, it is appropriate to use both of their values in the discounting equation. When using both $E(FV)$ and i_p in the discounting problem, the economist must articulate their independence and separable estimation, else risk will be doubly-counted in the resulting present value amount and the plaintiff is under-compensated for his or her losses.

Often the probability of η_{FV} is not thought of separately from i_p. For example, the risk free rate i might be 5% associated with U.S. Treasury Bills and the risk-adjusted interest rate i_p might be associated with a corporate bond rate of 9%. If the risk-free and risk-adjusted rates are known, we can compute the probability η_{FV} as follows:

$$\begin{aligned} i_p &= \left[\frac{(1+i)}{\eta_{FV}}\right]-1, \\ 1+i_p &= \left[\frac{(1+i)}{\eta_{FV}}\right], \\ \eta_{FV}(1+i_p) &= (1+i), \\ \eta_{FV} &= \frac{(1+i)}{(1+i_p)}. \end{aligned} \tag{34}$$

Using equation (34), in the example above, η_{FV} is 1.05/1.09, or 0.963. In this situation, the discounting equation can be written as either $PV = FV\left[\frac{1}{(1+9\%)}\right] = FV\left[\frac{1}{(1+5\%)}\right]\times 0.963$ or in the general framework as $PV = \sum_{t=1}^{n}\left(FV_t\left[\frac{1}{(1+i)^t}\right]\times \eta_{FV}^t\right)$.

In summary, if there is risk concerning the receipt of the future amount, the risk must be accounted for else the plaintiff will be overcompensated. The risk can be accounted for by adjusting the amount to be received to the expected amount or by adjusting the discount rate to reflect the risk (or both when independent risks exist). Equations (29) through (33) prove that the two common methods used to account for the risk of receiving the future amount are equivalent. In the case of independent risk, it is possible to account for some risk concerning the receipt of the future amount by adjusting the future value to account for some of the risk and to use the discount rate to adjust for the remainder of the risk. However, in all situations it is incumbent upon the economist to be able to explain the factors used in deriving the expected value and what factors were used in deriving the discount rate.

Inflation and the Discount Rate

Amounts to be received in the future may be a function of price inflation. To show how inflation affects the choice of the discount rate we begin with the explanation of the difference between a nominal discount rate and real discount rate. The nominal interest rate is the observable market rate. According to the Fisher[12] equation, the nominal interest rate is a function of the real interest rate and the expected rate of inflation. For discrete[13] growth rates, the function is

$$i = i_r + f + i_r f \,. \tag{35}$$

Where i is now equal to the nominal interest rate, i_r is the real interest rate, and f is the expected rate of inflation.

When discounting a future amount that changes according to the rate of inflation, the future amount can be inflated by the expected rate of inflation and then discounted by i. Conversely, the future amount, absent the expected inflation increase, can be discounted by i_r. We now show that these two methods of discounting are equivalent. When the future amount to be discounted changes with the rate of inflation the future amount would be:

$$L_t = L_0(1+f)^t \,. \tag{36}$$

where L_t is equal to the future amount and L_0 is equal to the current amount. To calculate the present value of L_t we substitute L_t into equation (26) so that

$$PV(L_t) = L_t\left[\frac{1}{(1+i)^t}\right], \tag{37}$$

where $PV(L_t)$ is equal to the present value of L_t. This method requires the forecast of inflation in order to calculate L_t from equation (36). Then, the nominal interest rate i must be chosen in order to calculate $PV(L_t)$ of equation (37). It should be noted at this point that by choosing i and f, the real rate, i_r, has implicitly been chosen from equation (35).

For the second method of calculating the present value of an amount affected by inflation substitute for L_t from equation (36) into equation (37), this provides

$$PV(L_t) = L_0(1+f)^t\left[\frac{1}{(1+i)^t}\right]. \tag{38}$$

Substituting from (35) into (38), rearranging, and canceling terms provides

$$\begin{aligned} PV(L_t) &= L_0(1+f)^t\left[\frac{1}{(1+i_r+f+i_r f)^t}\right], \\ PV(L_t) &= L_0(1+f)^t\left[\frac{1}{[(1+i_r)(1+f)]^t}\right], \\ PV(L_t) &= L_0\left[\frac{1}{(1+i_r)^t}\right], \end{aligned} \tag{39}$$

where i_r is calculated by rearranging equation (38) to

$$i_r = \frac{(i-f)}{(1+f)}. \tag{40}$$

The results above show the equivalence of estimating the present value by first calculating L_t from equation (36) and then using equation (37) to calculating the present value to using last equation in (39) where the i_r used is consistent with, according to equation (35), the f and i used in equations (36) and (37). Discounting a nominal future amount with a nominal interest rate is equivalent to discounting the real future amount with the real interest rate when the real interest rate is consistent with the nominal rates according to equation (35). It would be incorrect to discount a nominal future amount with a real discount rate or to discount a real future amount with a nominal discount rate. A calculation that discounts a nominal future amount with a real discount rate or discounts a real future amount with a nominal discount rate would indicate an unacceptable lack of economic understanding

or attempted deceit. The practical aspect of this equivalence is that it is not necessary to forecast a specific rate of inflation when discounting a future amount that will be determined by the inflation rate; it is necessary only to forecast i_r, the real interest rate.

Risks Associated with the Investment Vehicle

There are two types of risk associated with an investment vehicle that need to be taken into account when selecting a discount rate. One risk is the risk of default on the investment vehicle. To properly compensate an entity for a future loss, the probability of default on the investment vehicle must be accounted for. Let I_1 be the expected amount to be received in one year, that is, the amount that the entity requires in one year to be properly compensated. Then, the present value, from equation (26), is

$$PV(I_1) = I_1\left[\frac{1}{(1+i)^1}\right]. \tag{41}$$

The amount I_1 will be available only if the investment vehicle returns the initial investment and the interest in the amount of $PV(I_1)\times i$. The amount available would be

$$I_1 = PV(I_1)\times(1+i). \tag{42}$$

However, if there is some probability of default the expected amount available in period 1 will be less than I_1; the entity would be under compensated. The expected amount available in period one if there is a probability of default is

$$E(I_1) = [PV(I_1)\times(1+i)](1-p_d)+[0]p_d. \tag{43}$$

where p_d is the probability of default. If p_d is greater than zero the expected amount available in time period 1, $E(I_1)$, would be less than I_1. The expectation is that there is under compensation. (For simplicity this analysis assumes default means no money is returned, 0, the conclusion is the same however if default implies that a portion, less than one, of the investment and interest is paid at time period 1.)

The second risk is unanticipated inflation or deflation. This risk is an important consideration when the future value, *FV*, is a function of inflation. Unanticipated inflation (deflation) will result in the future value being greater (less) than expected at the time the present value calculation. And, as seen from equation (35), unanticipated inflation or deflation is not incorporated into the discount rate used at the time of the present value calculation. The result of unanticipated inflation (deflation) would be under

(over) calculation. The result of unanticipated inflation (deflation) would be under (over) compensation.

Evaluating the Selection of a Discount Rate

The generally accepted theory of the discount rate used to account for the time value of money includes the following components: the real interest rate; the expected rate of inflation; a premium for inflation risk; a premium for risk of default on the investment vehicle; and, a premium associated with liquidity and illiquidity of the investment vehicle. The discount rate is a function of all of these components. However, by various applications of the time value of money, one or more of the components of the discount rate might be assumed to equal zero. Nevertheless, the economic justification of the discount rate used must be grounded with an analysis of each of the above interest rate components.

The real interest rate must be positive; anticipated inflation can be positive or negative; and, the nominal interest rate can be positive or negative based upon the relative size of the anticipated inflation to the real interest rate. With regard to inflation's impact on the discount rate, the Fisher equation is generally accepted by economists. This equation states that the nominal interest rate is greater than the expected rate of inflation. Therefore, if the future amount to be received incorporates the inflation factor, the interest rate used in the present value calculation must be greater than the inflation rate used in that calculation. Or, if a real rate of interest is used for discounting purposes, the real rate of interest must be greater than zero. Economists use different methods to determine how much greater the nominal interest rate is compared to expected inflation, or, what the real interest rate is. The economist should be able to articulate the method used for determination of the interest rate.

If the risk involved with receiving the amount in the future is not calculated directly with an expected future value of the principal amount, the risk must be accounted for in the discount rate. And, if the risk is incorporated into the discount rate the economist must be able to divulge what portion of the discount rate is attributable to the risk of not receiving the amount in the future. In the case of independent risk, it is possible to account for some risk concerning the receipt of the future amount by adjusting the future value to account for some of the risk and to use the discount rate to adjust for the remainder of the risk. The risk associated with the investment vehicle has two components, the risk of default and the inflation risk. The expected future amount available will be less than the expected lost future value if the present value is calculated using a discount

rate that incorporates a risk of default. For this reason, it is necessary to use an interest rate on a risk free vehicle when determining the discount rate to discount an expected future amount to be received or directly increase the current amount to compensate for the risk of default. In all situations, when evaluating the discount rate, justification should be present regarding the risk factors that affect the likelihood of actually receiving the amount in the future.

Conclusion

We began this paper by stating that the purpose of this unified presentation of generally accepted economic principles is to (a) provide documentation for the common economic conclusions derived by forensic economists, and (b) advance the appreciation and acceptance of the science of forensic economics. After working through the generally accepted economic theories, the practicing expert should review his or her own methods to insure that they are grounded in generally accepted practices. In the same way, when examining an expert, the evaluator should have that person articulate his or her handling of each component of generally accepted economic theory presented in this paper to ensure appropriate and admissible expert scientific analysis is presented to the trier of fact.

Endnotes

[1] The wage rate in this context is the total unit cost of labor that includes money wages to the worker, fringe benefits, payroll taxes, workers and unemployment insurance, etc. The mix of money wages and other labor costs within the total unit cost of labor is variable under the theories of wage-rate determination. In most forensic economic wage rate determination problems, economists disaggregate their empirical analysis of money wages and fringe benefits. The 'unit' in "total unit cost of labor" is also variable and dependent upon the specification of the profit maximization function. The demand market for labor determines the units of earning capacity (e.g., a job giving a certain level of earning capacity might be constrained in the market to 8 hours per day for five days per week for 52 weeks per year). Due to market, mental, and physical constraints, a person may possess one or more levels of earning capacity. For example, within the labor market, a person might weekly supply H_1 hours of labor at C_1 level of earning capacity and also provide H_2 hours at C_2 level of earning capacity where $C_1 > C_2$. A typical situation for this example is that a person works a strenuous full-time day job for H_1 at C_1 and a light duty part-time night job for H_2 at C_2; it is unlikely that the person could work ($H_1 + H_2$) hours per week at C_1 level of earning capacity due to limiting market, physical and mental constraints.

[2] See Polachek and Siebert (1993) for a lengthy discussion of wage rate determination. The generally accepted neo-classical wage theory presented in this paper cannot in its entirety explain the labor market's operation. However, this wage theory represents the reliable core of the extended economic models of wage determination. For in-depth survey of the wage rate literature, see the references at the end of this paper.

[3] Wage rates usually lag behind prices in the short term due to sticky wage contracts or misperceptions of true relative price or price level changes; however, as the examined price/wage time period lengthens, the effect of lagging wages to prices is also of little consequence.

[4] Likewise, when $\partial R / \partial L < w$ firms layoff labor (e.g., the cost of labor exceeds the revenue it produces, so firms layoff portions of their workforce creating unemployment and thereby lowering the earning capacity of the laid-off workers).

[5] If the wage rate and the prices of all goods consumed by the worker rise by the same amount, the worker does not achieve any real economic growth.

Prices may not rise evenly and simultaneously across goods and wage adjustments may lag behind the rise in goods prices due to delays caused by wage negotiation.

[6] See Tinbergen (1939), p. 80.

[7] Whether past earnings losses are converted to present value by adding interest is usually a jurisdictional legal question.

[8] Utility is that property in any object or activity, whereby it tends to produce benefit, advantage, pleasure, good, or happiness, or to prevent the happening of mischief, pain, evil, or unhappiness.

[9] The importance of utility function being continuously differentiable (non-kinked) and strictly concave in optimization theory comes from the fact that if the function u is concave, then any point c at which first derivative $u'(c) = 0$ we find a global maximizer. Formally, let f be a function of many variables defined on the convex set S (a set of convex points is such that for any two points in the set, the line segment that connects them is also in the set). A set S of n-vectors is convex if $(1- \lambda)x + \lambda x' \in S$, whenever $x \in S$, $x' \in S$, and $\lambda \in [0,1]$. Then, f is strictly concave on the set S if for all $x \in S$, all $x' \in S$, and all $\lambda \in (0,1)$ we have $f((1-\lambda)x+ \lambda x') > (1+\lambda)f(x)+ \lambda f(x')$. Graphically, we say that f is a strictly concave figure if a line segment can be drawn so that it goes in, out, then back into the figure.

[10] The meaning of risk and uncertainty are often confused. Frank H. Knight established the economic definition of the terms in his landmark book, *Risk, Uncertainty, and Profit* (1921): risk is present when future events occur with measurable probability; uncertainty is present when the likelihood of future events is indefinite or incalculable. Uncertainty occurs in circumstances that cannot be analyzed on *a priori* grounds, because uncertain events are too irregular or are too unique to be observed through empirical observation. Since uncertainty is not measurable, it cannot be quantified through the interest rate, insurance, or other arrangements.

[11] As used here, the term "expected" is in the probabilistic sense. It is the product of the value of the event if it does occur and the probability of the event occurring.

[12] See Fama (1976), Hirshliefer (1970), and Varian (1978). Albrecht and Moorhouse (1989) provide a detailed discussion of the distinction between continuous and discrete growth rates.

[13] In this principle the discussion will be in terms of discrete growth rates as they are generally the rates that are used in forensic economics. The conclusions would be the same for continuous rates, see Albrecht and Moorhouse (1989). For a discussion of the minimal mathematical bias that occurs when the real rate is derived as continuous and presented as discrete see Brookshire and Smith (1990), Chapter 3, Appendix 3.

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